GEOMETRY AND ISIGORIA
THE GEOMETRICAL ARRANGEMENT OF URBAN SPACE AND THE CONNECTION WITH THE POLITICAL RELATIONS IN THE ANCIENT GREEK CITIES-STAGES

Nafsika Chatzidakis
Architect Engineer AUTH
MSc Urban and Regional Planning NTUA
PhD Candidate NTUA
nachatzi@gmail.com

Abstract

Mathematics is the language of nature, *but it is also the means by which we extract and understand the knowledge delivered by nature.*

*Mathematics and philosophy were born in ancient Greece as a result of the love the ancient Greeks harbored for precision of speech/thought and proof. Furthermore the Pythagoreans detected "numerous analogies between numbers and being", "... numbers were not only found in objects but also in attribute and everywhere…" and hence we believe that it is a logical consequence, the existence of **"… numerous analogies between numbers and cities"**. Furthermore, history tells us **that in Greece, from there very beginnings,** there was a close and multilevel relationship between city and mathematics.*

The present paper explores the relationship between "Democracy" and "Arithmetic and Geometry", and more specifically it refers to the geometrical arrangement of urban space and the connection with the political relations in the ancient Greek cities-stages (6th – 5th century b.C.). Further more it refers to the geometrical shape of circle and the way that it shows to us the meaning of isigoria (each one has equal freedom of speech), Isonomia (every body is equal for the law), and Isiokratia (every body has vote with equal mathematical value).

This study also refers to:

*.. Homer, who describes how the warriors would get in military formation **forming a circle within which each one has equal freedom of speech (isigoria)***

.. the mathematician Cleisthenes the Athenian and his democratic reforms based at the arithmetic analogies,

..the new urban design of cities during the 6th century where all public buildings are located geometrically and geographically around the "agora" (market in a way), the spatial center of the city..

*..to Ippodamos who chooses an open space and **carves straight roads intersecting at perpendicular angles, creating a squared checkered pattern city, α city of even squares, a city with the "center", which is the agora.***

Key words: *Geometrical shapes, Arithmetic relationships, numbers, analogies, cities, democracy, urban space.*

1. Mathematics

Mathematics is the language of nature, but it is also the means by which we extract and understand knowledge from nature. Mathematics is the reflection of the objective reality on the human mind, and even more is **absolutely necessary, for it is perhaps the only way** there exists, until this day, for us to perceive and to develop our environment with a scientific approach.

Mathematics (and more specifically such fields as geometry, analogy, perspective etc). have a long history of connection with Architecture and Urban Development. This relationship is uninterrupted; it is continuous from their beginning and in a sense is a codependent relationship. (note «Γεω-μετρία Geo-metry = *γέα-γη-earth + μετρώ-measure, **which was derived from the need to divide land and build/develop it.***".

Mathematics: "It is called mathematics, because it teaches us how we should learn things… Because within thought (intellect) there is their existence, and intellect alone is learning. Forms of mathematics: arithmetic, music, geometry, astronomy." (Giropoulou - Efstathiou A. 2009 p.445).

From the works of Heron the Alexandrian in his work Geometric Terms we learn the following: (Heron: Geometric Definitions)

«Where does mathematics derive its name from?

The wandering (philosophers) claim that even a layman can observe oration, poetry, and the other public musical disciplines, whereas when it comes to mathematics, one cannot begin to understand mathematics if one has not studied in depth. For this reason they named Mathematics the main discipline that deals with this matter (the main things that had to be learned).

It is believed that the Pythagoreans had given the special name (mathematics) only for geometry and arithmetic because previously they were named independently without some kind of nominal connection… and they named them thus because they found in both disciplines a scientific character and something worth learning. And this because they realized that they were dealing with that which is **eternal, unchanging and clear, and the only things which they considered as science.**

What is mathematics;

Mathematics is the science capable of creating theories **for connecting those things that are apparent with thought and senses.**

How many are the disciplines of mathematics

The most valuable and superior disciplines are two: Arithmetics and geometry

The disciplines that deal with the apparent (which are subordinate to the notional) are six: Accounting, Surveying - geodetics, Optics, Engineering, Astronomy, Musical Theory».

*****Note:

Heron the Alexandrian was an engineer and a geometrist. He lived in Alexandria of Egypt around the first century B.C. or the first century A.C. His most famous invention is the αιολόσφαιρα = aeolosfaira or ατμοστρόβιλος = atmostrovilos, the first steam engine in history. He was headmaster of the School of Fine Art of Alexandria, which effectively was the first polytechnic school ever established at the Museum for engineers. It is believed that he followed the theory of atoms and the Mechanical Syntax of Filon. Ideas of Ktisivious were a basis for some of his works. He was also know as Heron the Ktisivious (being probably a student of the great mathematician and inventor Ktisivious) and Heron the Engineer. (Loria G. 1972).

«Mathematics and philosophy were born in ancient Greece, as a result of the love the ancient Greeks had for accuracy of their speech and proof» (Berketis N. 2009).

Obviously, pre-Greek mathematics are mainly comprised off calculative techniques and arithmetic systems, which were related either with religion or practical matters, such as dividing fields and crops. **Greek mathematicians focused their attention towards accuracy of expressions and the strict logic of proof.** Furthermore, the science of mathematics was perceived by the Greeks as the main means of exercising their intellect and improving it.

«Legend has it that in 430 B.C., the residence of the island of Thilos, during a plague, consulted the Oracle of Delphi as to the solution of their problem. The oracles' answer mentioned that the plague would stop if the cubic shrine of Apollon in Thilos, was doubled in size while at the same time retaining its shape!!!» (Spanthagos E., 2008), (Loria G. 1972).

Here we see that the oracle, as the central point of existence of the ancient Greeks, expresses a mathematical problem to the people of Thilos for them to solve, because the application of thought to an object of higher quality, will yield as a result on the one hand the avoidance of suffering and the optimum solution to everyday matters, and on the other the elevation and improvement of their mathematical understanding, as mathematics is the means to practice ones intellect and evolve it!!!

This mathematical problem became known all over Greece and many mathematicians have applied themselves to solving it from Ancient times until even today. The impossibility of finding a solution to the problem of doubling the cube using only the ruler and the calliper has earned it the name "Dilemma of Thilos", which along with the «trisection of a random angle», and the «squaring of the circle», are known as the «unsolvable geometric excersizes».

Furthermore, it is common knowledge, that through history **the relationship of city-state and mathematics, at least in Greece, from Ancient times, has been very close and multi-layered.**

2. Arithmetic analogies in the Democracy of Cleisthenes

Starting reference to the relations of the political situation, in the Geometry and Arithmetic, the ancient Greek cities-states, let's refer to the relationship of «Democracy» and Mathematics, and to be more exact, to Cleisthenes the Athenian and his Democratic reforms.

Mr Economou G. in his study «Democracy and Geometry» concerning the specific matter mentions the following:

«The correlation of the great reform of Cleisthenes, which paved the road for democracy, with mathematics, was noted for the first time in 1925 by G.Glotz, who mentions the following: «Cleisthenes was an outstanding politician and at the same time a mathematician with a radical way of thought, which is expressed more specifically as a consequence of the Pythagorean dogma. there truly are certain elements, which can supports such an opinion:

α) firstly the political institutions set forth by Cleisthenes appear to give certain privileges to three specific numbers (3,5,10) and their derivatives. Three are the areas in which he divides Attica (city, mesogeios - middle earth = area and seafront). Thirty are the "trittyes", where a "trittye" defines the third of the tribe (the third, and the triad). Five hundred are the congressmen and fifty the chancellors of the governing tribe. Ten are the tribes and one hundred the municipalities.

β) the first Greek (refereed to as acrophonic or herodian) system of writen numbers (for which the scribe of the 2nd century A.D. who recorded and named the system, claims that he ascertained its use from one of Solon's laws), contains a clearly defined quintet and decimal character.

It is known that the Pythagorean theories attribute to these numbers mystical, existencial and political notions». (G.Glotz, Reference to Economou G. 1995).

Mr Vardiabasis, when referring to Clisthenes' reforms, notices that: «Clisthenes' technique is based on the decimal system. He divides Attica in more than 170 municipalities, of equal population, and from these municipalities, which work as minor local democracies, he forms the ten tribes, which before the reform were four.

The new ten tribes are divided in equally populated artificial «statewide» segments with no common geographical borders. Their territory is not continues. One third of the land and the houses of each tribe belongs to the city, one third to the middle lands and one third to the seafront. Hence, the tribe Akamantis for example will include one third, or a tritty of the residence of Keramikos (an area within the city) a tritty of the residence of Thorikos (Laurio – seafront) and a tritty of Sfittou (middle regions). Or the new Pandionis tribe which will include the trittyes Kithathinians (city) Paeania (middle regions) Mirinians (seafront) and so forth.

To what point one may ask? To have in the same governing body (the dome for example) the fifty chancellors of a certain tribe, i.e. the 16-17 congressmen elected by the people of the city with the same number of congressmen from the middle region and from the seafront.

These, fifty in total, a third very rich banker, ship-owners, craftsmen of the city, a third farmers and stock breeder from the middle region and one third sailors and fishermen from the seafront, would settle themselves successively every month in the «presidential mansion», in the dome they comprise the communal «Body of Parliament », and amongst themselves they elect, with beans, the daily «President of the Athenian Democracy». The fifty chancellors - congressmen times ten, as many as the tribes, comprise the Parliament of the Five hundred, which in the days of Solon had four hundred members. Apart from the parliament, every governing body had ten members. For example the Generals, headed by the Prime Minister, even the magistrates and the rest: treasurers, policemen, agronomists, and so forth. And 10 times 600 equals 6.000 which will be the Heliastites, divided in ten courts of justice of 500 members with 1.000 substitutes» (Vardiabasis N. 2013).

In the case of Cleisthenes, and in accordance with the affor mentioned references, since Cleisthenes was a mathematician of the school of thought of Pythagoras, he fashioned a different, much more elaborate for his time, perception of things, be they physical, mathematical or politically related. He had the philosophic -inquisitive- scientific consideration one could say, and as a result he studied politics and specifically relations of state, thus applying to the physical state the theoretical mathematical relations of analogy and balance. Once should bear in mind that the Pythagoreans applied themselves with the mathematical science and the relationship of numbers as well as with music and cosmology - astronomy conferring too these sciences the same mathematical relations.

«As with the relations of musical sounds and tune with the known arithmetic analogies of musical intervals of the octave, the quadrant (4/3) the firth (3/2) as well as the major (#), the difference between the quadrant and the fifth (9/8). (For the Pythagorians an interval is a straight segment). These analogies were proven by the experiments Pythagoras performed on the «single-string», a device with one string and one moving saddle which would allow only one part of the string to oscillate. Pythagoras divided the single-string in 12 equal interval (the same number as the vertices of the cube).

Pythagoras is the first philosopher who connects Astronomy with Music, claiming that in the harmonic and spherical universe everything is governed by laws which can be expressed with numbers of the «ιερής τετρακτύος=holy tetraktys». With the theory of the harmony of spheres, which combines the cosmic harmony with the musical harmony, the great philosopher tried to

explain the position and the motion of the planets. Using musical terms, i.e. euphonic musical intervals, he defined in the form a gamuts the interplanetary distances. Furthermore, by corresponding musical intervals to metric intervals, he was the first to calculate in Delphic stadia (unit of measurement used at the time) the interplanetary distances of the «ουρανίων γεννητών», as the uranium bodies were named by Plato who carried on the work of Pythagoras. Pythagoras, accepted in the 6th century B.C. that the musical distance between the earth and the moon is a single tune and the metric distance between them is 126.000 Delphic stadiae, i.e. 22.371.300 meters». (Spyridis Ch. 2013).

2.1. The timeless ternary faith of the Greeks

The pythagorians also believed that in principle the whole universe comprises **a totality in mathimatical order. Everything is divided in trinities.** Where the universe is concerned, it is divided in three worlds. All living things have a ternary nature: body, soul (which Pythagoras relates to mind) and spirit.

«...το παν και τα πάντα τοις τρισίν ώρισται...»

(according to the Pythagoreans), entirety and everything is defined by the number three,.... (Aristotle, Physics-On the Heavens, 286 α, 10-15).

Homer seems to be of a similar belief, claiming:

«…τριχθά δε πάντα δέδασθαι…» Everything is divided in trinities. (Homer Iliad Rhaps. Ο vers. 189).

Mr P. Adamakos (Mathematician), mentiones in his study «Homer's Mathematics and his ternary obsession» *concerning trinities*:

«Suidas mentiones Hermis as the **TRIS** megistus (three times great): «…διότι περί τριάδος εφθέγξατο, ειπών εν μια τριάδι μίαν είναι η θεότητα …», because for Trinity he was reported saying that in triplet there is god…" thus confirming the timeless ternary nature of the Greeks faith.

«Homer, a true fan of the Orphic-Pythagoral triad, used in his epics by choice only the number three and its multiples! For example:

«and **thrice** he (Diomides) wished with wrath for his (Aenia) demise, and **thrice** he was thrown by the God (Apollon) his shiny shield (Iliad. Raps. E, vers. 435), and **thrice** Patroklos climbed the walls and **thrice** he was smut down by Foivos, (Iliad. Raps. Π, vers. 702), and **thrice** roaring he (Patroklos) charged and **nine** (=3x3) men each time were sent to Hades… (Iliad. Raps. Π, vers.784). (Note: Sometimes he used four and ten but always in terms of the unknown to us Orphism)».

The Dactylic hexameter (δακτυλικός εξάμετρος) was used by Homer. We believe that, he follows this way of writing his poems with dactylic hexameter, in purpose, that means:

(The meter consists of lines made from six ("hex") feet. Each foot consists of 2 or more syllables that constituting the metric unit. In strict dactylic hexameter , each of those feet would be a dactyl (finger), **each dactyl is made of three syllables, one long and two short syllables, (− υ υ). The Homeric epics arrange words in the line so that there is an interaction between the metrical pattern (episode) - so the first long syllable of each foot - gives the natural, spoken accent of words. If these two features of the language coincide too frequently, they overemphasize each other and the hexameter becomes melodic - rhythmic).

Every verse is comprised of six dactyls (fingers), and each dactyl is made of three syllables, one long and two short syllables, (- υ υ). The two short syllable are sometimes substituted by a single long syllable (- -) In ancient Greek poetry rhythm is given by the ordered alternation between long and short syllable, in contrast with neo-Greek (modern-Greek) poetry in

which rhythm is given by the ordered alteration of intonated and toneless syllables. (From the Odyssey schoolbook 7th grade OEΔB, transl. D.N. Maronitis).

It is worth mentioning that Homer does not stop here! There are references which indicate that Homer, having a perfect knowledge of Mathematics, placed adjectives at specific positions within each verse, fourth foot, so as to form the golden section between the first parts and the remaining. There is also, in every verse, an eclectic small pause (no to be confused with the absence of *F*) in delivery, which was necessary for the vocalist, at the second foot, so as to result every time in a constant mathematical fraction! However, the trinity of the feet is an undisputed proof of the intentional perpetuation of the triumvirate substance!». (Adamakos P).

Finally Pythagoras also had a strong opinion when it came to political maters, as he dissuaded seditions, revolts and anarchy not only in his days but also in the days of the successors of his students. His teachings were retained for many generations. One of his many apothegms is: "in every chance we must drive away and cauterized with fire and iron and other inventions the sickness from the body, luxury from the abdomen, **rebellion from the state**, discord from the house and **from all together the lack of measure**".

3. The Mathematic Dimension of thinking of the Ancient Greeks during the 6TH Century B.C.

The aforementioned studies of the Pythagoreans give us a rough idea concerning the way of thinking of Cleisthenes, he was a politician and at the same time a Pythagorean Mathematician, and determines the way he acted reforming the political scene of the Athenians with the equivalent philosophical Pythagorean view.

Furthermore, for one to fully understand this "mathematical" dimension of political thought in Athens during the 6th century B.C. and its importance, and to form a complete opinion and deeper understanding of that age, one must first study and take into consideration the political and financial state, and far most the spiritual state of that era and its prevailing philosophy.

The truth is, that in those days great and "cosmogony" events were happening which affected the whole known world and especially the Greeks and their actions. It is in those days that we have the development of philosophy mainly by the Ionian philosophers, we also have the birth of scientific thought and the leaping development of mathematics, physics, astronomy, medicine, music and as a result the evolution of the arts (architecture, sculpture etc.)

As a result all these important evolutions affected the political establishment of the cities, and the general means of perception and reaction of individuals at the time, with a special emphasis on the city-state of Athens, being a city of prevailing cultural importance.

Of course, in order for one to understand fully the way in which this evolution took place, **one must also consider the opposite opinion,** which is to assume that this change in the way of thinking and the quest for truth, i.e. the scientific interpretation of "nature" and the principle questions of "being" and its "knowledge", are what brought about this social and political evolution and the Democratic political system and not the other way around. «**Democracy is the creation of the natural sciences of the pre-Socrates era** » according to M. Danezis and S. Theodosiou as mentioned during their television program on ET3 (Greek public television channel 3) «The universe I loved» (Danezis M., Theodosios S., 2013)

«The questions is not whether the city produces the geometry or geometry produces the city, but if Cleisthenian egalitarianism is in correspondence with the notion of the universe as expresses in the philosophy of physics by the Militians, which is to say whether the intellectual

world of Anaximandros could be understood by the founder of the new state and democracy». (Economou G. 1995).

That which Economou G. analyzes and supports in his article concerning Cleisthenes and the evolutions of the 6th Century B.C. is that «the spiritual and intellectual atmosphere towards the end of the 6th Century B.C. are characterized by a certain agreement between the geometric approach of the world, as established by Anaximandros and the other Militians, and the political approach of a rational homogenous state, as conceived by Cleisthenes».

In general the 6th century B.C. witnesses a change in the cosmological perception of the Ancient Greeks, followed by a different perception of space and the world and «Vernant explains: the Ancient Greeks, being the founders of cosmology and astronomy, have given them a certain direction which has defined the evolution of these sciences, and the entire Western history … and.. Greek cosmology was liberated from religion, and the knowledge concerning nature lost its wholly character, because exactly the same time, social life was rationalized and the governance of state became to a greater extend, a "cosmic", i.e. a none-religious, activity».

3.1. Geometry and State

In Jean-Pierre Vernant's study «Geometry and Spherical Astronomy in the first Greek cosmogony» the following are mentioned concerning the relationship between «geometry and state»:

«The Ionian physicists - Thales, Anaximandros, Anaximenis - focus on presenting, through their cosmological works, a theory, a point of view, a general perception that can explain the universe without any religious implications, without any mention of the divine or holy practices. In contrast, the physicists had full knowledge that in certain aspects of their beliefs they were in full opposition to the traditional religious beliefs of the time.

How should we explain this turn towards astronomical thought, this intellectual change? One of the experts in ancient astronomy writes: «Babylonian astronomy is purely arithmetical, whereas Greek cosmology is from its very beginning geometric... The only explanation that I can find for this is that the Greeks are born geometrists». However, this explanation seems somewhat lacking. I would like to purpose another. Between the days of Isiodos and Anaximandros lots of changes took place, both on a social and financial level. Their importance has rightly emphasized many times. As for me, I would like to stress the point that in my opinion is the most important when it come to understanding this change we are dealing with: **This change, in my opinion, represents a political phenomenon, it is <u>the birth of the Greek city state.</u>**

We are trying to explain a certain astronomical perception of the universe, **we are therefore dealing with a pattern of thought based on the conscience and well processed stochastic levels.** This pattern of thought is expressed through a specified vocabulary and is organized around certain basic concepts; it is presented like a coherent and structured system of notions. This vocabulary, these basic concepts, are things new compared to the past. **To understand how they were formed, we must first investigate the form in which the changes of social life on a conceptual level were expressed.** In another words, we must identify the private life sector which played the intermediate role between the notional structures and the renewal of certain superstructures. To find this intermediate link between the Greeks' social practices and the new notional universe we must investigate how the Greeks of the 7th century B.C., when faced with the crisis of an expanding sea trade and the first steps towards a currency based economy, adopted a new way of thinking concerning their social life, trying to reform it in accordance with certain desires for equality; how, in this way, did it become a subject of contemplation and how was it transformed into meanings.

For the first time in human history, it would appear, that **a certain level of social life is separated and becomes the subject of a particular investigation, of an intentional contemplation. The state's institutions do not dictate the existence of a "political" sector, but also a "political thought".** The term "τα κοινά = commons", which indicates the political sector, mean that which is the same for all, in public affairs". (Jean-Pierre Vernant 2011).

Truly, in human life, there are for the average Greek two very distinct sectors, one private, family, domestic sector (that which the ancient Greeks called economy, οικονομία = οικο + νομος = house + law/order) and one public sector which included all the decisions of communal interest, anything that alters the community in a closely united group, and converts this group in town. When the ancient Greeks said "the city" meant the citizens and not the city space we mean today.

"Within the frames of the city-states institutions (the city state that appeared after the days of Isiodos and during the age of Anaximandros), nothing that belongs to the public sector can no longer be regulated by a single individual, even if that individual is the king. Amongst those that comprise the political society, all matters concerning the common sector are discussed freely and openly within the "agora", a public discussion, which takes the form of an actual debate. **Hence the state requires a specific procedure, one that removes the spiritual character from social life and rationalizes it.** The king-highpriest who abiding by the religious calendar does that which must be done on behalf of the human group and for the human groups well being, ceases to exist..

Henceforth the people hold in their hands their "common" fate, and they decide after discussion.

For the people, matters of the state can only be settle after public discussion, during which each individual can freely intervene to develop ones own argument. **Logos (=speech/reason/logic =>meanings in Greek language), the means by which these public discussion take place, has two meaning for Greeks: on the one hand we have words, the <u>speech</u> delivered by the orators at the (εκκλησία του δήμου) = "people's political gathering"; and on the other hand we have <u>logic</u>, the ability one demonstrates to reason with arguments, an ability which defines humans as not just a living being, but as "political being", a logical being".** (Jean-Pierre Vernant 2011).

We have an epistole to Ferekidis, which Diogenes the Laertios attribute to Thales (Diogenes Laertios 1,43). Ferekidis was contemporary to Anaximandros and the first - as is maintained by many of that time – to publish work in prose. Thales rejoices for Ferekidis's wise choice not to keep his knowledge to himself, but to bestow it upon the community, «ες το ξυνόν» (κοινόν) = "to the public". Hence it became a matter of public discussion. Philosopher Ferekidis **converts private knowledge to a subject of discussion,** similar to that which takes place concerning political maters. Like political cases, knowledge, discoveries, the same way every philosopher's theories concerning nature, will be intended for everyone, will become communal.

"I believe that Greek **cosmology managed to break free from religion,** and knowledge concerning nature lost its divine character **because during the same period, social life itself was rationalized** and the governance of the state became, to a greater extent, a "cosmic", that is to say a non- religious, act.

We should not stop here. Except for the rational positive approach to astronomy, we must also investigate its content and examine its origin. How did the Greeks form this new image of the world? **As stated previously, that which characterizes Anaximandros' universe, is his well rounded point of view, his sphericity.** The prime value for the circle for the ancient Greeks is well known. For them it represents beauty, the perfect shape. Astronomy must explain the phenomena, or according to the more traditional formulation to "save the phenomena", constructing geometric shapes, such as the movement of all the stars form circles. We must now

ascertain that the political sector is presented as being closely connected with a representation of space, which demonstrated the importance placed on the circle and the center, giving them a very well specified significance. From this point, it can be said that the appearance of the city is marked firstly by the transformation of the space of city, namely urban design. In the Greek world, and firstly probably at the colonies, we come across **a new town plan, according to which public buildings are concentrated around the "agora".** The Phoenicians were merchants, traveling with their ships the entire Mediterranean Sea, many centuries before the Greeks. The Babylonians were also merchants, who developed trade and banking techniques, much more sophisticated than those of the Greeks. Neither the Phoenicians, nor the Babylonians had an "agora". **The existence of an "agora" implies a social life system, which requires subjecting to public discussion all matters of the state.** That is why we come across "agora" only in Ionic and Greek cities. **The existence of the "agora" is a sign of the appearance of the cities' political institutions.** (Jean-Pierre Vernant 2011).

What is the historic origin of the "agora"? The "agora" has of course a certain past. It is related with certain characteristic habits of the Greeks. In Homer, we find the expression "λαόν αγείρειν", which means literaly to gather the army / the people. The warriors are gathered in military formation: **they form a circle.** Within the circle, which has thus been formed, is created a space for public discussion, within which each one has equal freedom of speech, **isigoria.**

In the beginning of part B of the Odyssey, Telemachos calls the "agora" to convene, i.e. calls together the military aristocracy of Ithaca. As soon as the circle is formed Telemachos proceeds to stand in the middle, takes the scepter and speaks freely. When finished, he exits the circle and someone else takes his place and retorts. **This assembly of "equals", consisting of gathered warriors, delineates circular space with a specific center,** where each and everyone can freely speak whatever he pleases.

Hence the human group forms the following image of itself: Apart from the individual, the private houses, there is **a "center" where public affairs are discussed, and this center represents all that is "communal", the community as a commune.** Within this center, all are equals, no one submits to no one. Within this free discussion, which takes place in the center of the "agora", all citizens are defined as **equals, as alike.**

"We witness the birth of a society, in which **the relationship between individuals is defined as a relationship of identity, symmetry, reversibility.** The human society does not form, as the mythical world, a world of **different levels,** having the king at the top and underneath an entire hierarchy of social situations, dictated by relations of domination and submission; now, the world of society is made up of relations of equality and reversibility, and all citizens are defined, one related to another, equal on a political level. One can say that the citizens, having the ability to enter **in this circular space of the "agora" with the specified center, at the same time enter in the framework of a political system, that is governed by balance, symmetry and reciprocation.**

In order to better understand the relationship between the political institutions of the state, we must look at individuals like Ippodamos the Militian. Ippodamos lived one century after Anaximandros, but shares the same current of thought. What is his field of specialty? It was assigned to Ippodamos to rebuild Militos after its destruction. He rebuilt it in accordance with a holistic design, which indicates his intention of **rationalizing the city area.** Instead of an Archaic type city, which can be compared to the medieval cities, with a labyrinth of streets sloping downwards and lacking order on the sides of a hill, Ippodamos choose an open space, **he carved straight streets which intersect at right angles, a checkered city, with even squares blocks, centered around the "agora".** It is often said that Ippodamos is an architect, the first great urban

planner of the Greek world. But Ippodamos is first and far most a theorist of politics, one who grasps **the arrangement of city space as an element, amongst many, of the rationalization of political relations.**

He is also an astronomer who deals with "meteorology", one can understand that in the same person **co-exist the astronomical avocation, that involves the heavenly sphere, the search for better political institutions and the attempt to build a city according to an orthological, geometric standard.**

The comedian Aristophanes presents a second example. In his play "ὄρνιθες" (chickens), he presents on scene, to ridicule, an astronomer, Meton, who had achieved as is well known, based on a chronological calculation, to relate the monthly lunar calendar to the yearly solar calendar. Aristophanes depicts him surveying the city and saying: "And I will measure with a straight ruler which I will apply, and thus the square shall be transformed to a circle, in the center will be the "agora", and there in the center will lead equal straight roads, it will be like a star, which despite being circular, its rays will radiate straight in every direction". These words trigger the cheers of Pistheterous: "This man is equal to Thales!". At this point it is obvious that Meton is trying to solve the problem of squaring the circle. He claims to be designing a circular city, with its roads intersecting at right angle, and at the same time converging at its center. The streets must intersect at right angle, because that is the logical, but at the same time, all streets must intersect at the center, because at the center of every human city exists the "agora" and because every human group consists a circle of sorts. One must also note Meton's astronomical observations, such as for example the suns rays: something obvious for an architect who is also an astronomer". (Jean-Pierre Vernant 2011).

The word Agora prodused from Greek verb «agorevo = αγορεύω» that means deliver a speeches in public gathering.

Deliver a speech in public orate, perorate, harangue.

Thus, the market requires a city with political relations that constitute democracy. In this way, agora presupposes cities with political associations than create the Democratic constitutions.

It should also be noted that in this era of major changes in philosophy and the social, economic and political relations in the Greek cities, the Hippodamian system of urban space, relates the geometry with the political relations and it rationalizes them. The Philosopher Hippodamos at the same time is studying and constructing a city with straight streets and squares blocks, and he rationalizes the urban space and political relations. Secondly, creating the spatial center of the city market (Agora), creates an imaginary center of the urban area (circle), which refers to political equality. The people come into an area centered (territorial), and simultaneously entering into a political system where the relationship of the citizens is the equality in the political level (mentally).

4. The Geometric Perception and the Holiness of the Circle for the State

In the above mentioned study by Vernant, the parallelism between the equal distance of the centre of the circle from all its periphery points– and a."Isonomia" = "Egalitarianism (=equality against the law)", b. "Isokratia" = ("Equality" of vote), and c. "Isigoria" (=equal right of speech)", between the population that forming a circle and discuss, is observed.

Many times the forms of the geometric shapes are attributed to both animated and non-animated objects, but they are also attributed to meaning, for example "history goes round in circles", "square logic". The symbolism of geometric shapes can be found and refers to many levels. The semicircle and the circle with their lack of corners refer to the sky, the brain which is of

spherical shape and also the trail of thought within the brain. The Orthodox church is built having domes on its upper part forming circular shaped (spheres, semicircles etc) which represent the heavenly. Even in ancient Greece burial site were constructed with circular, semicircular, hemispherical and cylindrical shapes, hence their names "tholotos tafos"(=domed/vaulted tomb) The main difference being that depending on the religious discipline of the time the curved shapes would symbolize either the heavenly powers or those of the underworld, but never the earthly.

The circle goes so far as to refer to the caress, the circular motion involved to caress/cuddle. The mother's hug is also a circle. If we wish to further understand the deeper subconscious effect of the circular shape, **the meaning of "love", the "three-stranded" complete love for ourselves – other – the world, could also be described with a circle.**

Even the shape of bread, having geometric properties, has a result which is related to its shape. From the same dough different shapes of loaf have different tastes.

Life, its essence, everything that has form and is included inside it, is based on geometry and numbers. With this "logos" (=reason). Mathematical reason is it possible to perceive diversity and infinity. Nothing is the same, everything is unique and infinite like the numbers.

Hence, the circle is an "archetype" shape, it is a holy shape symbolizing harmony and perfection, including amongst other symbolisms that of equality, at the same time being a mathematical shape with all the points of its perimeter having an equal distance from its center. When the circle is formed by the people present at the assembly, in accordance with the political relations, it symbolizes the "equality – egalitarianism – isigoria" of all those present, who with their bodies form a basic geometric shape. He who wished to speak, would stand in the center of the circle, where he would voice his opinion as an equal amongst equals (having an equal distance spatially and socially) and then anyone who wished to answer would in his turn take up the position in the center of the circle voicing equally freely his opinion concerning the subject. Each speaker is at an equal distance, and equal to all other attendance at the perimeter of the circle.

The three basic principles that characterize the true meaning of the Athinian democracy where **Egalitarianism, Equality and Isigoria, which symbolically are given by the shape of the circle**

Isonomia - Egalitarianism, *Meaning the equality of all against the law and the equal and just treatment of all by the justice system, but also the way in which laws are passed and who's interests they have in mind*

Isokratia - Equality *meaning that the votes of all citizens have the same mathematical value and strength.*

Isigoria *meaning that every citizen has the right to take the stand and address the people, so the people may decide upon that which he proposes.*

Anyone who has visited Pnika (=Pnyx) will have read and have seen that every citizen has the right to rise to the stone podium and talk to the people gathered at the opening.

The circle and sphere hold a very important and holy position in the Greek world since the beginning of its existence. Plato exalts the circle and notably the sphere calling it "πάντων τελεώτατον ομοιότατον τε αυτό εαυτώ σχημάτων", which means the most perfect and whole, in correspondence to the wholeness of the universe, and the most even shape, and furthermore declaring that the even is a thousand time more beautiful than the dissimilar («μυρίω κάλλιον όμοιον ανομοίον») [Πλάτων, Τίμαιος 33b]

The military bevy – circle in Homer's time **will later evolve after several economic and social changes into the cities "agora",** where all the citizens will be able to discuss and jointly decide about the affairs that regard them collectively. Once again the center (both spatially and politically), of the society of the city is the "agora", where the "holy" and the "commons" reside,

as Ippodamos teaches us. The "holy" and the "commons" are the main and the important, and for this reason their buildings are constructed with exquisite brilliance as opposed to private residence.

The idea of the sanctity of the circle and its center are so deeply rooted in the Greeks philosophy as mentioned above, that we come across it even in their private residence where they have a central holy center of the notional center of the residence (geometrically and functionally), that is the holy "estia"=hearth. There, at the "estia", a stranger may stand and beg requesting assistance from the master of the house, who in return may not deny him as is stated by the basic priciple of "Xenios Zeus"=The God of hospitality. (Homer tells us that Odysseus tries with the help of Nafsika to enter the palace and reach its "estia" so that the king Alkinoos, when he identifies him, will not be able to deny him his assistance.). (Homer, Odyssey).

Another related event that can be cited in that which is told by Irodotos:

When the tyrant of Samos Polykratis died, Maiandrios, the indicated successor, adopts the democratic ideas and refuses to accept authority. He thus calls a (circular) assembly in the "agora". He gathers within this "circle", this "center" of the human community, all the citizens to inform them that he disapproves of Polykratis who ruled his equals as a tyrant and under the circumstances, decides to **"deposit"** his authority (the state) into the **"center"** (amid), i.e. to restore to the people's community that which had been usurped by one individual, and to declare **"isonomia" (democracy).** (ΗΡΟΔΟΤΟΣ 3,142).

Epiloge: «There is a vast number of analogies between numbers and state».

«The word «Δίκαιον=Just/Justice/Righteous» comes from «δίχα» - according to Aristotle - and means half and half: the existence of equal portions for all.

So what do the ancient Greeks do? They place the judges seat, the speakers podium and the contestants' trophy in the center of a cirle of equal citizens. The equal distance between each individual and the center of the circle, the circle's radius, they name Metron (unit of measurement), thus mitigating each other's vanity, extravagance, each ones hubris towards another and to come to terms with one another based on their humanity. "Πάντων χρημάτων μέτρον άνθρωπος", they used to say». (Vardiabasis N. 2013).

«Πάντων χρημάτων μέτρον άνθρωπος»,= man is a measure of truth and knowledge. (Protagoras)

We ascertain a "significant" influence of the geometric perception in the cosmology, in the sciences in general, in the arts, in the city, in politics and so forth, and we identify the Greek geometrical "logos" (=speech, analogy, reason, logic) which penetrates all boundaries, and establishing communication between all levels, making them thus more concordant and interdependent elements of the whole, of the total, of the "universe".

Hence, during the 6th century BC, Greek cerebration escapes the religious cosmological perception and as a result we have the interaction of politics and mathematical laws. This cast of mind is abandoned at the end of the 5th century BC. It is then that the Philosophers, the Geometrists and the Astronomers were segregated from the state (they separated the city), and thus we also have the trials for contempt. It was then that the state and democracy go through a crisis with the known from history results and consequences (civil wars etc).

During these times Plato will stand by his "Politia", putting the political problem on a new basis. He rethinks equality and the relationsip between individuals and regimes, lifestyle, virtues and mainly justice. He redefines such concepts as politics and organization of the state and the relationships of its citizens both in meaning and as to how they are applied. He defines the «αρίστην πολιτείαν=ideal state» and prescribes as ideal governers "philosophers" scientists **setting**

as a prerequisite for them the knowledge of the sciences of arithmetic, geometry, music, astronomy, oration (Plato, Πολιτεία VII 521d, 540c).

Besides, Plato considers numbers as immortal and unchanging beings which, together with ideas, exist in the world of "being" (word of ideas and forms)" and not word of "becoming (material world of change)" of the mortals, and this is also confirmed by Aristotle who says that "because those investigators (philosophers) who accept the ideas - forms, match them to numbers" (Aristotle, Μετά τα Φυσικά 1072, 25-30).

We agree with G. Economou, who tells us that:

«Hence geometry defines the act of birth of democracy and its dying theory, under different form of course, based on the Cleisthenian affirmation in the first case and the Platonic reversal the second» (Economou G. 1995).

We are also support that: Geometry and Arithmetic are the birth of democracy at the great reform of Cleisthenes, constitute the maturity of the democracy at Plato's "Politia", and finally recommend the death of democracy with the removal of the philosophers and the separation Philosophy and mathematics from the city.

Today scientists are not philosophers, and the city is not governed by Philosophers-scientists, city is not organized by the basis of Mathematical Philosophy.

Based on the abovementioned testaments we can infer that with these remarkable parallelisms in vocabulary, in meanings, in logical structure and in analogies between state - politics and geometry - mathematics, they seem to validate our opinion about the closely interrelated relationship of mathematics and the state, and more specifically our opinion that:

The state and human society, with regards to its institutions and structure, should be formed and shaped itself resembling by analogy to the applicable geometric and mathematic cosmic laws, thus ensuring isonomy - isocratia – isigoria, the balance, the analogy, the justice and the harmonious "logos". We presume that in the opposite case the state passes to a state of "chaos" meaning disorder and gap, just how the present era involving a great global economic and social crisis is formed.

In light of the above, bowing to the quotes of our ancestors, who ascertained "**numerous analogies between numbers and beings**", and since the city is known to be an entinty (=being), we believe that there it is natural and it is expected that:

"**There exist numerous analogies between numbers and cities**".

References

➢ Adamakos P. (2013), *The mathematics of Homer and his triadic obsession,* article, [accessed 20-02-2013] <http://users.sch.gr/padamakos/page1.html>
➢ Vardiabasis N., (2013), *Cleisthenes,* article. [accessed10-02-2013] <http://www.ekivolos.gr/i%20athinaiki%20dimokratia(eleftherotipia).htm>
➢ Graviger P., (1998), *Pythagoras and the secret teachings of Pythagorism,* Athens: Dimeli Press
➢ Danezis M., Theodosiou S., (2013), *The Universe I Loved,* broadcast Greek tv channel ET3, [04-03-2013 2:30]
➢ Berketis N., (2009), *The influence of mathematics to Philosophical evolution of Plato on education and Universe,* article, [accessed 20-01-201] <2lyk-ymitt.att.sch.gr/mfilos2012.pdf>
➢ Oikonomou G., (1995), Democracy and Geometry*, "Hellenic Philosophical Journal",* Vol.12, 36, September 1995
➢ Spandagos E., (2008), *Delian problem in Ancient Greece,* Athens: Aithra Press

➢ Spyrides C., (2013), *Pythagoras,* article, [accessed 19-02-2013], <http://www.inews.gr/87/to-pythagoreio-theorima-tou-sybantos.htm>
➢ Tziropoulou - Efstathiou A., (2009), *The meaning of words,* p.288,328,331,425,612, Athens: Georgiadis Press
➢ Glotz G. *Histoire Grecque,* Paris 1925, refer to Oikonomou G., (1995), Democracy and Geometry*, "Hellenic Philosophical Journal",* Vol.12, 36, September 1995
➢ Vernant Jean-Pierre (2011), *Geometry and spherical astronomy in the first Greek cosmogony,* [accessed 16-02-2013] <http://grmath.blogspot.gr/2011/07/1-jean-pierre-vernant-jean-pierre.html>
➢ Loria Gino, (1972), *History of Mathematics,* transl. Koveos M., Athens: Hellenic Mathematical Society 1972
➢ Aristotle, *Metaphysics,* (1002, 30-35), Athens: Kaktos Press (Ancient Greek Secretarial "The Greek) 1991
➢ Aristotle, *Physics,* Athens: Kaktos Press (Ancient Greek Secretarial "The Greek) 1991
➢ Aristotle, *On the Heavens*, Athens: Kaktos Press (Ancient Greek Secretarial "The Greek) 1991
➢ Herodotus, *Histories,* Athens: Georgiades Press (Library of Greeks), 2007
➢ Heron, *Terms of Geometry and Geometrics,* Author: Kipouros C., Athens: Hellenic Mathematical Society 1995
➢ Homer, *Iliad,* trnsl. Kazantakes N., Kakrides I., Athens: M.Rodis Press, 1964
➢ Homer, *Odyssey,* trnsl. Kazantakes N., Kakrides I., Athens: M.Rodis Press, 1964
➢ Plato, *Timaeus,* (52a,b,c,d, and 37d,e), trnsl. Grypares I., Athens: Zacharopoulos I. Press (Ancient Author's Library) 1998
➢ Plato, *Republic,* trnsl. Grypares I., Athens: Zacharopoulos I. Press (Ancient Author's Library) 1998